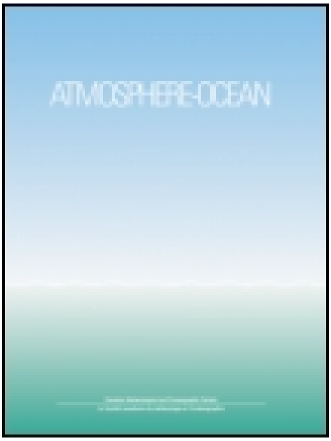

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On the growth rate of wind-generated waves

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On the Growth Rate of Wind-Generated Waves

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ABSTRACT *A new approach to fetch-limited wave studies is taken in this paper. Using data from five towers arranged along a line from the eastern shore of Lake St Clair, the differential growth between towers is explored as a function of local wave age. It is argued that this method avoids the usual fetch-limited pitfall of inhomogeneity over long fetches and, in particular, the changes in wind speed downfetch of an abrupt roughness change. It is found that the growth rate decreases uniformly downfetch as the waves approach full development. This differential method leads to a smooth transition from rapidly growing short fetch waves to the asymptotic invariant state of full development. When the variation in wind speed after an abrupt (land to water) roughness change is taken into account, the idea of a universal fetch-limited growth curve is called into question.*

RÉSUMÉ *On utilise une nouvelle approche pour étudier les vagues limitées par le fetch. À l'aide de cinq tours installées le long des rives est du lac Sainte-Claire, on examine l'accroissement différentiel entre les tours en fonction de l'âge des vagues locales. On pense que cette méthode évite les problèmes habituels de non homogénéité limitée par le fetch sur de longues distances et, particulièrement, les changements de la vitesse du vent en aval d'un changement soudain de la rugosité. On constate que le taux d'accroissement diminue uniformément en aval du fetch lorsque les vagues approchent de leur développement maximum. La méthode différentielle entraîne une transition régulière des vagues de court*

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fetch qui grandissent rapidement au stage asymtotique invariable d'un développement complet. Lorsqu'on tient compte de la variation du vent après un changement soudain (rivage à eau) de rugosité, on s'interroge sur l'hypothèse d'une courbe de croissance universelle limitée par le fetch.

1 Introduction

The study of the development of wind-generated waves in response to an offshore wind has been an important element in the more general problem of predicting the evolution of waves on open water bodies. It is generally accepted that beyond a certain critical wind speed (see, for example, Kahma and Donelan, 1988) ripples near the shoreline give rise to longer and larger waves as one moves offshore. This monotonic development continues until, at large enough distances offshore, the phase speed of the largest wave components exceeds the wind speed and the net rate of development approaches zero. The resulting asymptotic state of "full development" is difficult to demonstrate from field data because the wind is seldom uniform over the long distances required and usually wind measurements are only available for a few locations. Nonetheless, the pioneering study of Pierson and Moskowitz (1964) and more recently that of Ewing and Laing (1987) and the weight of common experience have enshrined the concept of full development in our picture of waves at sea. At the other end of the development scale, the early growth of waves advancing from an upwind shore is far better documented both in the laboratory (Sutherland, 1968; Mitsuyasu, 1968, 1969; Hidy and Plate, 1966); and in the field (Burling, 1959; Hasselmann et al., 1973; Kahma, 1981; Donelan et al., 1985; Dobson et al., 1989). A consistent framework in which to view these fetch-limited waves has been provided by Kitaigorodskii (1962) and has generally been employed in the study of wave growth with fetch. Kitaigorodskii (1962) has argued that, under conditions of stationary and homogeneous offshore wind, suitably non-dimensionalized characteristics of the surface gravity wave field can be specified by the three parameters: fetch, x; friction velocity, u_*; and the gravitational acceleration, g; or the non-dimensional grouping of these $\tilde{x} = xg/u_*^2$, the non-dimensional fetch.

The existence of these two regimes of wave development ("fetch-limited" – fetch-dependent growth; and "full-development" – no fetch dependence) implies a transition region between them. However, all the fetch-limited studies yield power law dependencies of wave energy on fetch so that full development may not be achieved at sufficiently long fetch. Linear fetch dependence such as in Hasselmann et al. (1973) shows no tendency to approach full development, whereas weaker than linear dependence (e.g. Donelan et al., 1985) suggests an approach to full development in the limit of infinite fetch. Our expectations are rather more along the lines of the development curves of Bretschneider (1973) based on the early work of Sverdrup and Munk (1947) in which the transition to full development from fetch-limited conditions is via arbitrarily chosen analytic functions. There is a clear need for suitable field observations to establish the transition from fetch-limited to fully developed waves.

There is, however, a daunting list of reasons why such field observations have not yet been reported. Perhaps the most significant reason lies in the unrealistic assumption of stationarity and homogeneity over the time- and length scales required to achieve full development. At typical wind speeds of 10 m s^{-1} these are, respectively, of the order of 16 h and 200 km. Quite apart from the problem of observing the wind at sufficiently frequent intervals* over the entire fetch, the difficult question remains about how to treat a non-stationary or inhomogeneous wind within the structure of the Kitaigorodskii similarity hypothesis. Unless the offshore wind direction is exactly normal to a straight beach, the direction of wave propagation will depend on the adjacent shoreline geometry (Donelan et al., 1985). Another source of inhomogeneity in the wind arises through the change in roughness when the wind leaves the shore to develop a new boundary layer over the much smoother water (Taylor and Lee, 1984). This question has been looked at in another fetch-limited study (Dobson et al., 1989) and recommended there that the linear average over fetch (of the time average over the record) be used as the appropriate scaling variable. Since fetch-limited wave energy scales roughly with the second power of the wind speed it is not at all clear that linear averages are appropriate or even that the time average of the wind speed is relevant rather than, say, $\left[\overline{U^2(t)}\right]^{1/2}$.

Janssen et al. (1987) have suggested that growth curves scale more closely with u_* than with U, the mean wind speed at 10-m height. However, in such fetch-limited studies, the friction velocity is generally deduced from the measured wind speed via some empirically determined drag coefficient rather than directly measured. This calls into question the accuracy and applicability of the drag coefficient formulation to the particular circumstances of acquiring the wave data.

A further problem associated with the study of the development of wind waves is the effect of swell on the wind sea. Donelan (1987) has suggested that the well known reduction in the energy of laboratory wind waves when mechanical (swell) waves are added is due to the swell-induced detuning of the resonance conditions for quartet non-linear wave-wave interactions. Similar effects have also been observed in Lake Ontario in the unusual circumstance of the coexistence of sea and swell. However, Dobson et al. (1989) find good agreement of their growth laws from the swell-infested Atlantic off Nova Scotia with those derived from swell-free data on Lake Ontario (Donelan et al., 1985). The question of the effect of swell on wind sea growth is far from resolved. In the meantime it behooves us to seek a consistent set of observations in a swell-free environment.

The establishment of an accurate fetch-limited "law" is of such fundamental

*An estimate of a suitable interval may be deduced from the scale of separation of mesoscale and microscale fluctuations in the marine atmospheric boundary layer – "the spectral gap". The microscale fluctuations arise through direct interaction of the low-level wind with the boundary and thus may be parametrized via the wind and such boundary-layer characteristics as roughness and thermal stability. The mesoscale fluctuations, on the other hand, arise through larger scale planetary boundary layer dynamics and may not be deduced from the observed surface wind at a point. Therefore, in order to track the mesoscale variability, the wind observing stations should be separated by a distance of the order of the mean wind speed/frequency of the spectral gap. Pierson (1983) has reported observations of horizontal wind spectra that place the spectral gap at about 0.0008 Hz, so that in a typical mean wind of 10 m s^{-1} the appropriate spacing of observing stations is about 12 km.

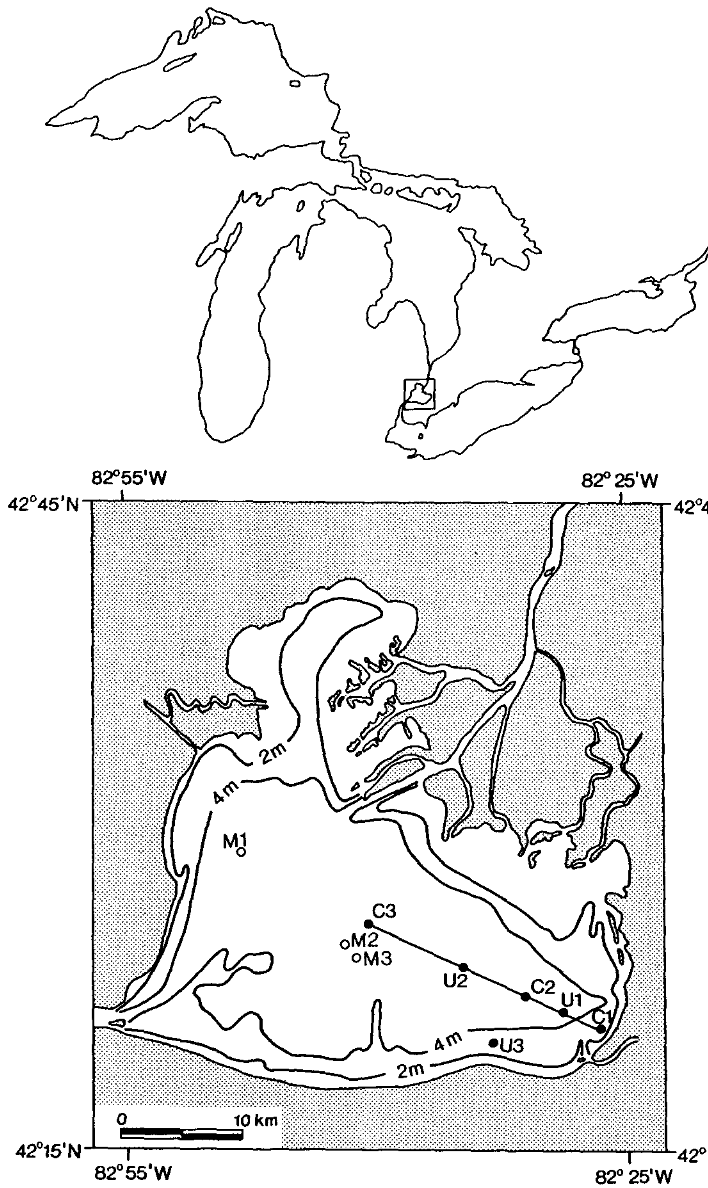


Fig. 1 Map of Lake St Clair, located within the Laurentian Great Lakes system, showing the location of towers (C, U) and buoys (M).

importance in assessing the fidelity of numerical wave prediction models (see, for example, SWAMP GROUP, 1986) that a fresh look at the problem is warranted.

These ambitious goals were part of the motivation for an extensive field experiment on Lake St Clair in the fall of 1985. This work attempts to use these wave and wind data to realize the twin goals of establishing an accurate fetch-limited growth law and the approach to full development.

TABLE 1. Biases and rms differences of individual buoys from the average

	Wind Speed		Wind Direction	
Buoy No.	Bias ($m\ s^{-1}$)	rms ($m\ s^{-1}$)	Bias (°)	rms (°)
1	−0.10	0.49	7	19
2	−0.45	0.51	−4	11
3	0.10	0.48	−1.5	10

2 The experiment

During 1985, scientists of the National Water Research Institute (NWRI) of Environment Canada and of the Great Lakes Environmental Research Laboratory (GLERL) of the (U.S.) National Oceanic and Atmospheric Administration, set up six wave observing towers on Lake St Clair (Fig. 1). Five of the towers were aligned along a bearing of 295° True – this bearing being in the most likely direction of the expected intense fall winds and also being normal to the eastern shore. The sixth tower lay near the southern shore. Three towers were equipped and operated by NWRI and three by GLERL. These are designated C and U respectively in Fig. 1. The NWRI towers supported three capacitance wave staffs arranged in a right isosceles triangle of 25-cm adjacent sides and were equipped with either a cup anemometer (towers C1 and C3) or a wind vane (tower C2). The data were digitized at 4 Hz and recorded in situ. The tapes were changed weekly and, to avoid exhausting the limited data capacity, recordings were made only on even hours and then only if the mean wind exceeded a preset threshold. Each recording lasted 20 min or 250 wave periods of the longest waves expected. The GLERL towers supported a single Zwarts (1974) wave gauge and the instantaneous surface elevation was transmitted via frequency modulated radio link to shore, where it was then digitized at 4 Hz. Thus the NWRI towers yielded some information on wave direction and on wind speed or direction, but the records were discontinuous and no very light wind cases were recorded. The GLERL towers yielded continuous wave information and that only. These different approaches allowed us to study many different aspects of wave growth and dissipation.

Three meteorological buoys, each consisting of a moored toroidal float with an attached instrument tower, (designated M1, M2 and M3 in Fig. 1) provided the data on the wind field. The buoys measured wind speed (scalar average), wind direction and air temperature at 4 m above the water surface, and water temperature just below the surface. All data values were averaged over the 20-min wave recording period each hour. The design accuracies of the sensors were 1 $m\ s^{-1}$ for wind speed, 5° for wind direction, 0.5°C for air temperature and 0.2°C for water temperature. Hourly data from the available buoys were then averaged to provide a single mid-lake value of each measured parameter for each wave recording period. In order to check the validity of using a single average wind, we compared the buoys individually with the average of the buoys (up to three) reporting in any hour. The biases and rms differences of each buoy from the average of the buoys are given in Table 1. The rms differences include the biases and are about 0.5 $m\ s^{-1}$ and 15°.

These are comparable to the design accuracies and it was considered acceptable to use a lake-wide average of all reporting buoys. This ensured that there were wind data to compare with wave data in all cases. The wind speeds (towers C3 and C1) and direction (tower C2) reported by the NWRI towers were useful only as a check on the buoys since their reporting scheme was conditional on even hours and high winds. The GLERL towers, on the other hand, had no wind information but reported wave information continuously.

In order to remove the influence of atmospheric stability on subsequent analyses of wave growth parameters, all wind speeds were converted to an equivalent neutral 10-m wind speed by the following procedure. For each hour, the wind speed, air temperature, and water temperature data were used to determine wind friction velocity (u_*), Monin-Obukhov stability length (L), and surface roughness (z_0), by means of the profile method described in Liu and Schwab (1987). This method uses the Charnock relation $z_0 = \alpha u_*^2/g$ along with dimensionless stability functions for stable and unstable conditions as given by Long and Shaffer (1975). Then a 10-m wind speed for a neutral wind speed profile that produces the same friction velocity u_* as the actual profile was determined. This 10-m wind speed is used in all subsequent calculations. Liu and Schwab (1987) show that normalization of wave parameters using the profile friction velocity u_* (or the 10-m equivalent neutral wind speed, since it is just a constant times u_*) determined by this procedure effectively eliminates the influence of atmospheric stability on wave growth curves.

The 20-min average values of 10-m equivalent neutral wind speed, wind direction and air-water temperature difference are plotted in Fig. 2. After day 307, wind speeds frequently exceeded 10 m s^{-1} and reached 20 m s^{-1} on day 336. The air-sea temperature difference ranged from 8 to −11°C.

The use of mid-lake winds for all the towers would lead to a systematic bias for towers C1, U1, C2 and possibly U2 in an east wind, since the wind speed increases as the air blows from land (rough) to water (smoother). Dobson et al. (1989) have demonstrated the importance of correcting for this bias, and we adopt the same procedure of applying the Taylor and Lee (1984) guidelines to calculate the expected values at different fetches from the mid-lake measurements. We applied the procedure to each 20-min average taking into account the changes in fetch with wind direction. For an easterly wind the upwind land surface is flat and consists largely of marshes, to which we assigned a roughness length of 0.07 m. Westerly winds, on the other hand, are assigned a roughness length of 1 m appropriate to flow over upwind urban areas (Oke, 1978). The overlake roughness is deduced from the relationship between roughness length and wave parameters given by Donelan (1990). The wave parameters are calculated at mid-lake from the JONSWAP relations (Hasselmann et al., 1973).

Dobson et al. (1989) recommend the use of a linear average of the wind speed along fetch rather than the wind speed at the point of wave observation. We believe that the root-mean-square (rms) wind speed is more appropriate since wave energy scales roughly with the square of wind speed (Hasselmann et al., 1973). Consequently, for each case at each tower we computed the Taylor and Lee corrected winds for ten linearly spaced points along the fetch, and used the rms of these for the wind speed.

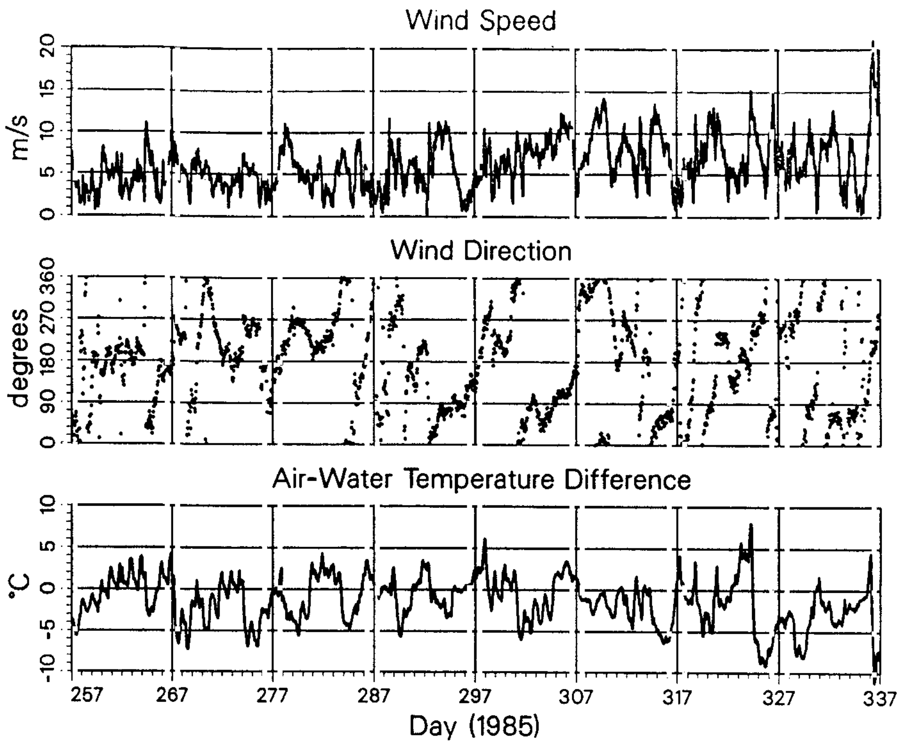


Fig. 2 Average meteorological conditions throughout the experiment: autumn 1985.

Application of the Taylor and Lee guidelines in this way led to a substantial reduction in the easterly wind speed at C1 and progressively smaller changes at the other towers, with that at C3 being essentially unchanged from the mid-lake average. The net effect of employing these fetch-corrected winds was a significant improvement in the correlation of wind and wave parameters when all towers are considered together.

3 Analysis

a *Wave Spectra*

The spectra for the waves measured at all towers were computed in the same fashion. The time series from the U.S. towers were processed at GLERL and those from the Canadian towers at NWRI. Before routine analysis began, spectra were computed on the same time series by both groups and various subjective aspects (e.g. detrending, windowing) of the analysis programs adjusted so that the results were identical. By using exactly equivalent algorithms to process the data, we could be certain that small differences in the results were indeed real and not the result of the method.

The recordings were of 20-min duration, sampled at 4 samples s^{-1}. The resultant spectra are the average of four spectra each computed from 1024 scans, thus using a total of 17 min 4 s of each recording. The frequency increment for the spectra is

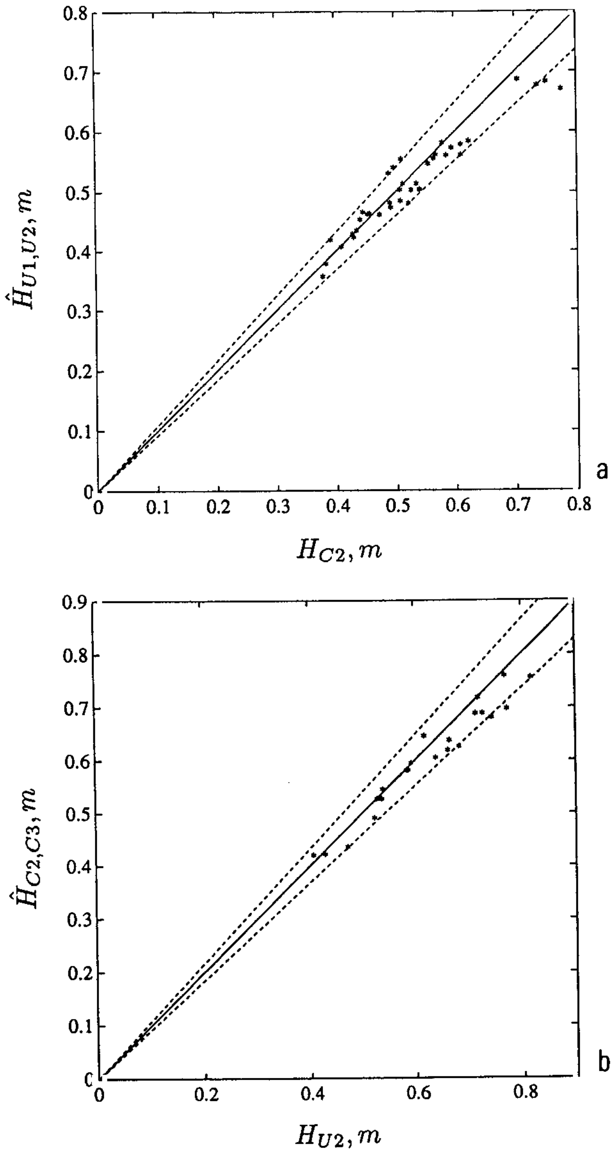


Fig. 3 Comparison of significant height measurements between U.S. and Canadian Towers: (a) height interpolated between towers U1 and U2 ($\hat{H}_{U1,U2}$) vs height at tower C2 (H_{C2}); (b) $\hat{H}_{C2,C3}$ vs H_{U2}; (c) $\hat{H}_{C1,C2}$ vs H_{U1}.

0.03125 Hz, so that the degrees of freedom for a spectrum with no missing data is 64. From the spectra, the characteristic wave heights were computed as four times the square root of the variance. The peak frequency was determined in the following way: the frequency associated with the largest spectral component was found; then using that spectral component and the values of the spectral components on either side of it, the peak frequency was defined as the resultant centroid.

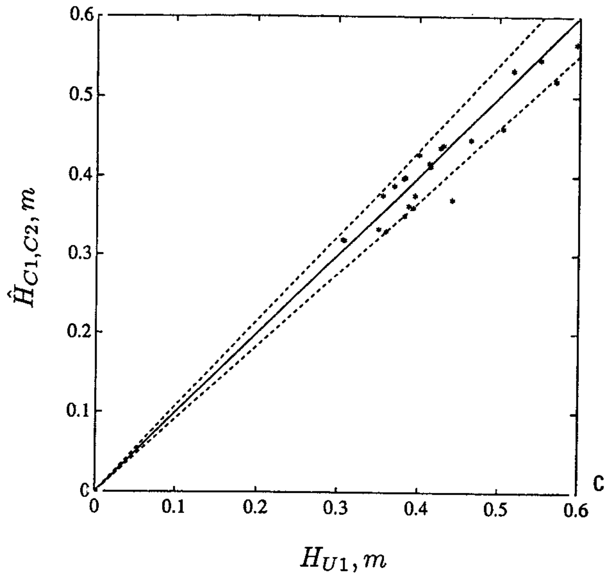


Fig. 3 (*Concluded*).

Such spectra have 90% confidence limits of 0.75–1.33 of the true value, and the expected coefficient of variation of the peak frequency estimates is about 2.5%, and the 90% confidence limits on the significant height are about ±8% (Bishop and Donelan, 1988).

In order to check the consistency of measurements by the two very different measuring systems (U.S. and Canadian), we examined cases in which the wind direction was within ±30° of the line of towers. In every case where three adjacent towers reported data we have plotted the measured value at the middle tower against the interpolated value from the outer towers to the location of the middle tower, assuming the energy is linear in fetch. Agreement 90% of the time within the 90% confidence limits of the 45° line suggests a close correspondence between the measuring systems. Such agreement is seen in Figs 3a and b; the agreement is only slightly worse in Fig. 3c. Similarly, 90% of the peak period estimates (Fig. 4) are within 2 standard deviations of the mean. Here the interpolation was proportional to $x^{1/3}$. Evidently the five towers, U1, U2, C1, C2 and C3, are a consistent set and permit the acquisition of meaningful differential measurements among themselves.

b *Differential Growth*

Attempts to establish fetch laws for wave energy and frequency have not yielded consistent results. Among the reasons for this, given in the introduction, two appear to be prominent, namely, the variation in wind speed along fetch generally caused by the abrupt change in roughness from land to sea (Taylor and Lee, 1984; Dobson et al., 1989); and the differences in wave and wind propagation due to fetch geometry (Donelan et al., 1985). Both of these effects are most pronounced close to shore, so that direct application of scaling methods to wave properties, based on fetch to the point of observation and on an average wind, are likely to be distorted to a

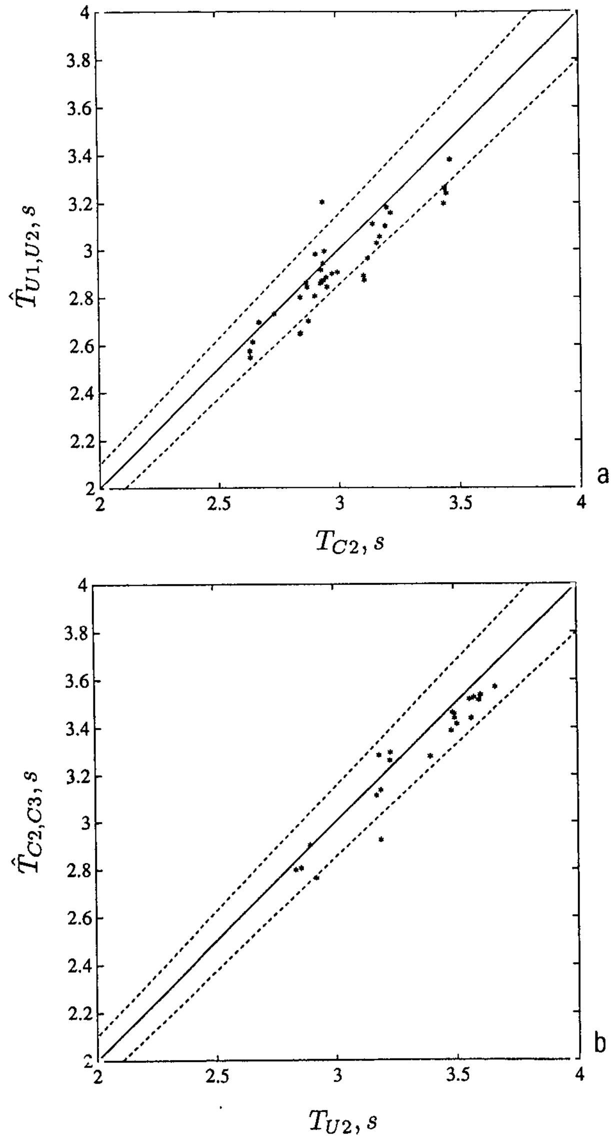


Fig. 4 Comparison of peak period measurements between U.S. and Canadian towers: (a) period interpolated between towers U1 and U2 ($\hat{T}_{U1,U2}$) vs period at tower C2 (T_{C2}); (b) $\hat{T}_{C2,C3}$ vs T_{U2}; (c) $\hat{T}_{C1,C2}$ vs T_{U1}.

greater or lesser extent depending on the degree of roughness change, complexity of shoreline geometry and fetch of the observing point or points. Furthermore, there is virtually no possibility of observing full development in such a manner since adequate horizontal homogeneity over the full fetch simply does not occur.

In the following, we use the multiple observing stations in Lake St Clair to circumvent these difficulties by looking at differential growth between stations rather

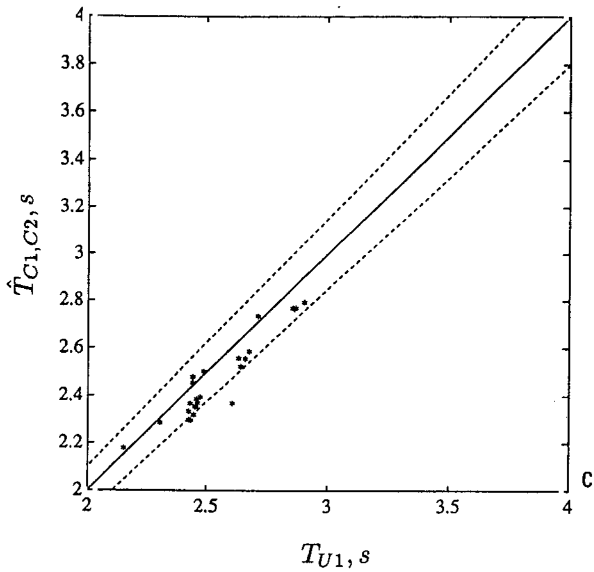


Fig. 4 (*Concluded*).

than overall fetch-dependence. From the differential growth versus wave age "law" and the excellent local correlation between non-dimensional energy and wave age, we can reconstruct the fetch-dependent growth of waves in an offshore wind. Furthermore, the time and fetch variation in mean wind may easily be accommodated in the differential approach we take here.

We consider only cases in which the mean wind speed exceeds 3 m s^{-1} and the mean wind direction lies within $\pm 30°$ of the line of towers. For consistency, we chose to use the wind direction, which is available all the time. Since the line of towers is nearly normal to both east and west shorelines, we would not expect large differences in wave and wind directions for the restricted data within $\pm 30°$ of the line of towers. Both east winds (85–145°) and west winds (265–325°) are considered. Since we are interested in the deep-water fetch laws, only cases in which the depth-to-peak wavelength ratio exceeds 0.3 are admitted. The noise level due to quantization and telemetry on the U.S. towers was estimated to be about 0.02 m; therefore a variance of 0.0004 m^2 was subtracted from the U.S. data. The noise level on the Canadian towers from instrument noise and quantization errors was less than 0.001 m and therefore negligible. A lower limit on the wave heights was placed at 0.02 m to avoid excessive errors due to quantization and telemetry noise.

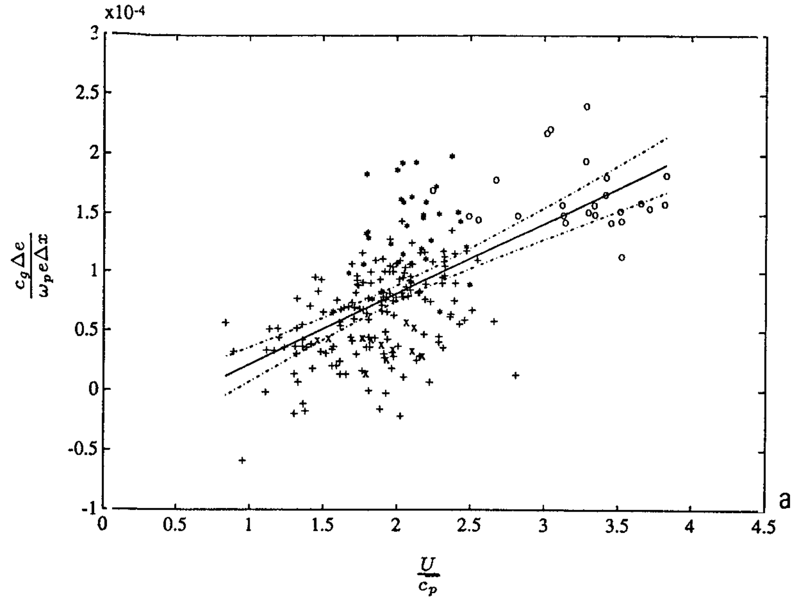
It is well known that the amount of wind input to surface waves, as reflected in the fractional energy increase per radian, is related to the ratio of wind speed to wave phase speed (e.g. Donelan and Hui, 1990). In the overall energy balance of a deep-water wind sea, other processes such as non-linear wave-wave interactions and wave dissipation are also important (Hasselmann et al., 1973). These are closely related to the wave steepness, which itself is proportional to the inverse wave age U/c_p (Huang et al. 1981) where c_p is the phase speed of the waves at the spectral peak. Inasmuch as all the dynamic components of the energy balance (the so-called source/sink functions) are closely related to the wind speed to wave phase speed ratio, it is reasonable to expect the overall growth rate (i.e. the increase of surface elevation variance) to be dependent principally on the inverse wave age U/c_p. Accordingly, in Fig. 5a we have graphed the overall fractional energy increase per radian, $c_g \Delta e/\omega_p e \; \Delta x$ versus U/c_p. Here c_g is the theoretical group velocity at the peak (radian) frequency ω_p, e is the variance of surface elevation ("energy") and Δx is the fetch difference between adjacent towers, i.e. the distance between them multiplied by the cosine of the angle between the wind direction and the tower line. Note that in deep water the theoretical (linear) group velocity $c_g = g/2\omega_p$ where g is the acceleration due to gravity. The arithmetic mean of tower pairs of the e and ω_p values are used in (1), and the corresponding c_g is calculated from it.

There is a great deal of scatter in the points of Fig. 5a (correlation coefficient = 0.62) but this is to be expected, given the $\pm 8\%$ confidence limits on the significant height (about $\pm 16\%$ on the energy) at each location and the expected difference, Δe, between locations. For example, if the fetch-limited growth of energy is proportional to the nth power of fetch, as in all published growth curves, then $\Delta e/e = n\Delta x/x$. Using the linear growth rate of JONSWAP (Hasselmann et al., 1973) $\Delta e/e$ is about $\frac{1}{2}$ on average for east winds (Fig. 1) and less than $\frac{1}{5}$ for west winds. This, coupled with the $\pm 16\%$ confidence limits on the energy at both towers of a pair, means that the 90% confidence limits on the expected value of the difference are 0.36 and 1.64 for east winds and far wider for west winds. Only data for which $\Delta x/x$ exceeds $\frac{1}{2}$ are included in Fig. 5a. This criterion eliminates the west winds and 12% of the east wind cases. The symbols identify the tower pairs used for each observed fractional energy increase. The solid line is the linear regression considering individual data points and the dashed lines are the expected 95% confidence limits of the regression.

The scatter of the data is a consequence of the appreciable sampling variability of the estimates of wave energy, compounded by the relatively small fetch differences between tower pairs. Both of these sources of variability can be reduced, but only by increasing the averaging time of the energy estimates and the fetch difference between towers. Doing this, however, would place stricter requirements on the stationarity and homogeneity of the growth process than are achievable with real data. This is the classic dilemma in analysing geophysical data, accentuated here because we seek to estimate a differential property of the process. We are limited, therefore, to datasets that are short in time and compact in space.

In spite of the scatter, we know with the probability of 0.95 that the true regression line falls within the hyperbolae shown. The point of "full development" corresponds to the value of U/c_p where the growth line changes sign. The wave energy at full development depends rather sensitively on this. The point of full development ($U/c_p = 0.83$) determined by Pierson and Moskowitz (1964) has been verified by other long-fetch studies, e.g. Ewing and Laing, 1987†. With this point fixed, two limiting straight lines fall within the 95% confidence limits (Fig. 5b):

†Ewing and Laing published 33 cases of which 14 were believed to be steady and homogeneous. The average U/c_p from these 14 was 0.86 ± 0.10.



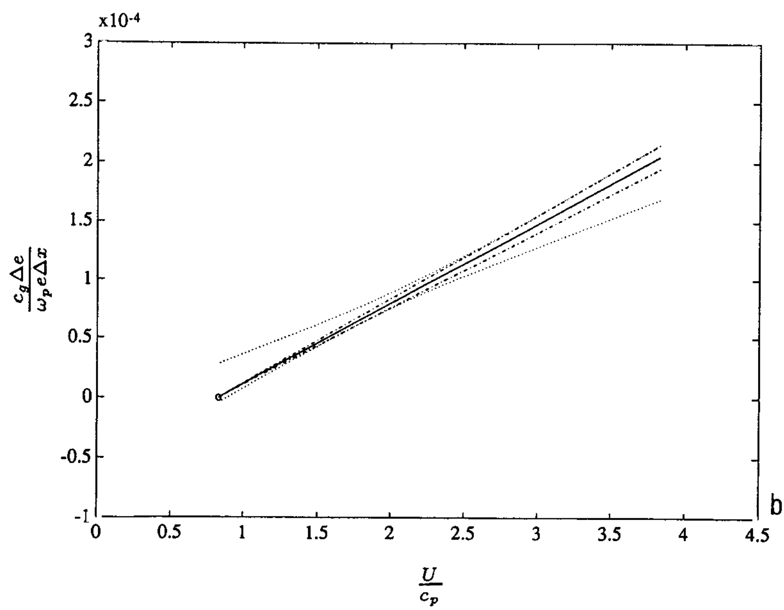


Fig. 5 Normalized differential growth versus U/c_p: (a) individual data points: U2 to C3 (×); C1 to U1 (○); U1 to C2 (*); U1 to U2 (+). The linear regression (—) and 95% confidence limits (–·–) are also shown. (b) linear regression (—) and 95% confidence limits (· · ·) of (a) are shown. The curves defined by zero growth at $U/c_p = 0.83$ and tangent to the two confidence limits are also shown (–·–). The zero growth at $U/c_p = 0.83$ is indicated (○).

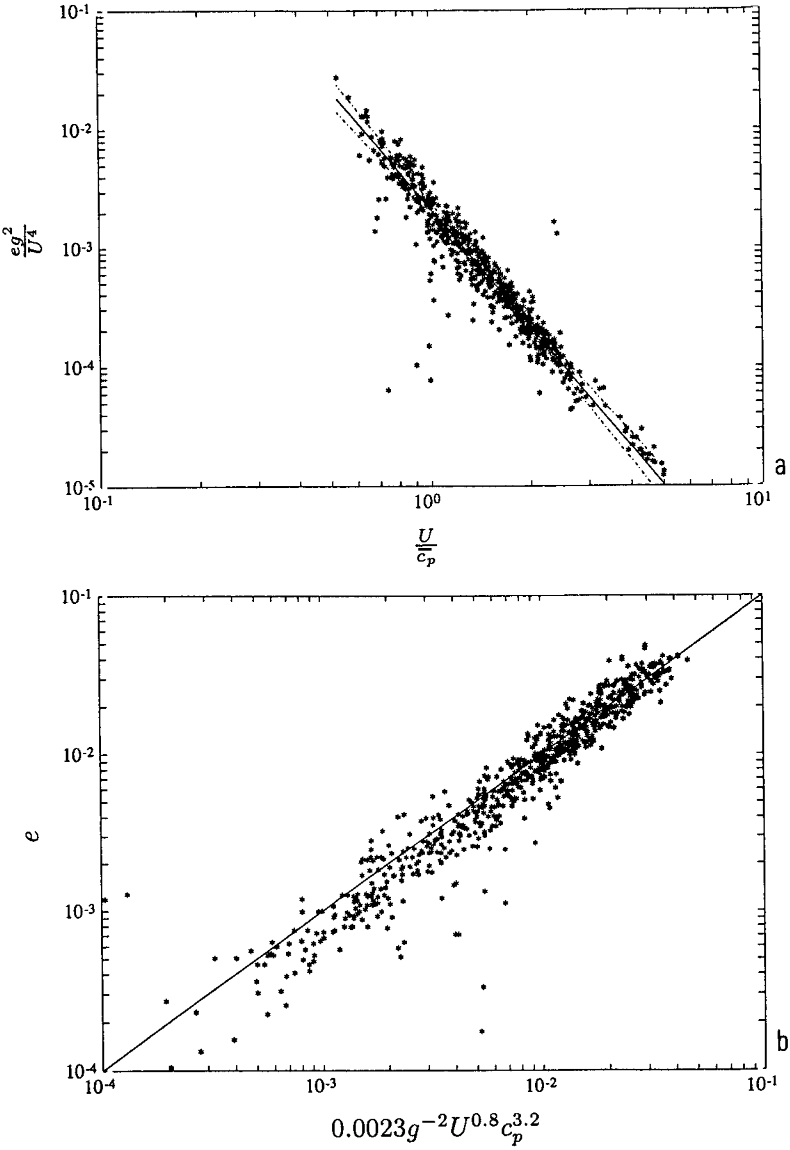


Fig. 6 Dependence of wave energy on wind speed and peak phase speed: (a) non-dimensional form with resulting regression line (—) and 95% confidence limits (–··–); and (b) dimensional form using the relationship deduced from Eq. (3b).

$$\frac{c_g}{\omega_p e}\frac{\Delta e}{\Delta x} = A\left(\frac{U}{c_p} - 0.83\right) \tag{1}$$

$$A_{max} = 0.72 \times 10^{-4}; \quad A_{min} = 0.65 \times 10^{-4}$$

The determination of fetch-dependent growth, $e(x)$, from the differential growth rate (1) requires a relationship between e and U/c_p. This is best expressed in the non-dimensional coordinates of Kitaigorodskii (1962): $\tilde{e} = eg^2/U^4$ vs U/c_p. All the data have been plotted in this way in Fig. 6a. Apart from a handful of outliers, the data are tightly clustered about the regression line. The range of U/c_p is wide – extending from full development to about 5.8. There are more data points in this plot than in the differential plot of Fig. 5a, because not all time slots were filled by all towers, thereby reducing the number of differences. At first glance it may seem curious that there is no tendency to "saturation" of these data near full development. However, it should be noted that this is not a fetch diagram and there is no requirement that approach to full development should produce a levelling off of the non-dimensional energy. Over the entire range the points lie along a consistent line, which is well represented by the fitted regression line:

$$\frac{eg^2}{U^4} = 0.0022 \left(\frac{U}{c_p}\right)^{-3.3} \tag{2}$$

with the correlation coefficient −0.95. The maximum likelihood regression line has been determined using rms errors in U, e and c_p of 8, 16 and 4%, respectively. The wind speed variability is taken from the observed rms differences between buoys divided by the mean speed. The sampling variability in the wave parameters is determined in the manner described by Bishop and Donelan (1988). The 95% confidence limits for this regression line are also given in Fig. 6a.

The appearance of a common variable, with some sampling error, on both ordinate and abscissa, raises the possibility of a spurious correlation (Kenney, 1982). Maat et al. (1991) have suggested that the effects of possible spurious correlations may be checked by rearranging the non-dimensional form of the equation into dimensional form so that no variable appears on both sides. A high correlation of these dimensional variables is taken as evidence that the original non-dimensional regression line is not greatly affected by spurious correlations. In our case, this is a highly subjective and very insensitive test, since quite large changes in the exponent of U/c_p in (2) still yield high correlations.

We take a somewhat different approach here. The dimensional form of the energy equation (such as (2)) may be written without assigning a specific value to the phase speed (or wind) exponent:

$$e = \alpha g^{-2} c_p^{\beta} U^{4-\beta} \tag{3a}$$

This form does not suffer from spurious correlation effects and the data may be used to determine α and β. Here β was determined by requiring that the correlation between the logarithms of the left- and right-hand sides be a maximum. (The wave variance, e, covers nearly three orders of magnitude so that the use of linear variables would weight the results too heavily toward strong wind and high U/c_p.) Once β was determined, α followed from minimizing the squares of the differences between the left- and right-hand sides of (3a). The resulting dimensional relation for e versus U and c_p is

$$e = 0.0023 g^{-2} U^{0.8} c_p^{3.2} \tag{3b}$$

with correlation coefficient of 0.94, and is shown in Fig. 6b.

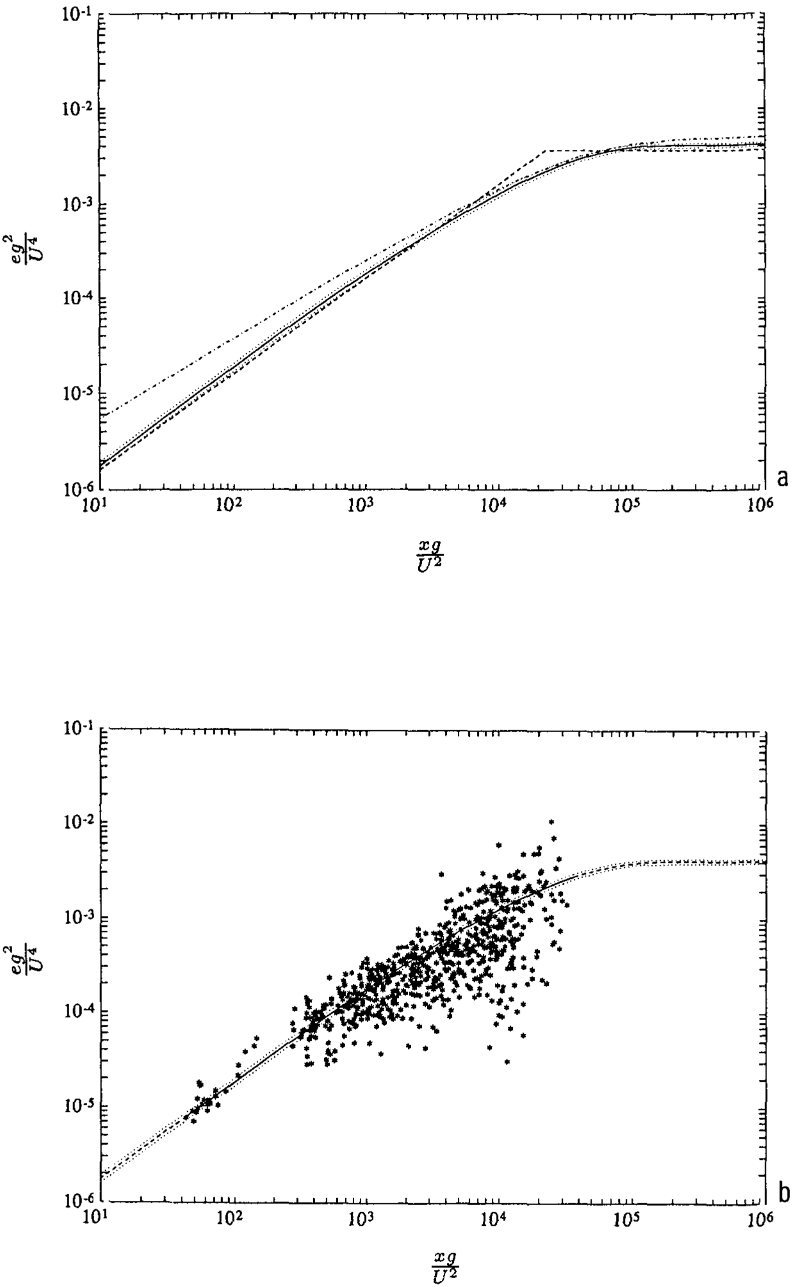


Fig. 7 Non-dimensional fetch dependence of wave energy. (a) Eq. (5a) (—) with 95% confidence limits (· · ·); JONSWAP with a Pierson-Moskowitz full development limit (– –); SMB (–·–). (b) Eq. (5a) (—; extrapolated: – –) and the 95% confidence limits (· · ·) showing the relation to these data (*). Eq. (5a) (—) showing the relationship to the JONSWAP regression line (– –) and the JONSWAP data (*).

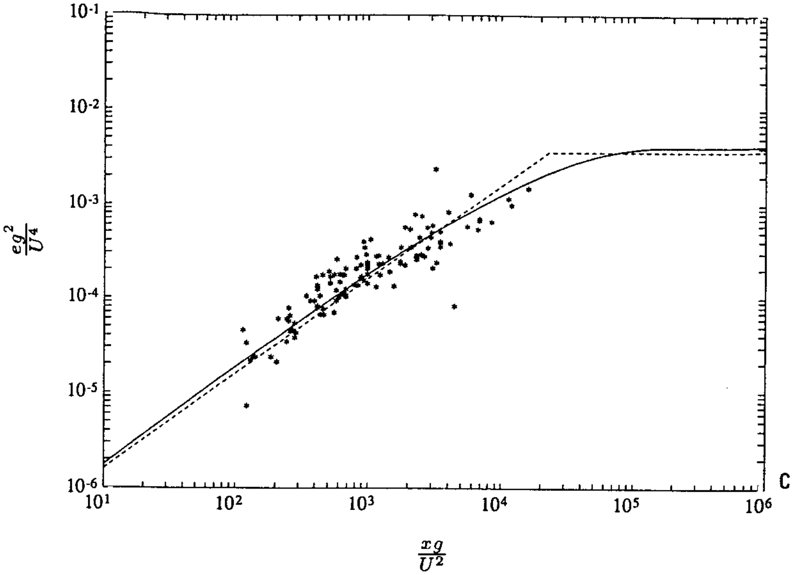


Fig. 7 (*Concluded*).

A further check on the validity of the procedure was carried out by dividing the 762 data points into three equal parts and repeating the procedure. The values of α and β changed in the third significant figure only.

Equation (3b) may be reorganized in non-dimensional form:

$$\frac{eg^2}{U^4} = 0.0023 \left(\frac{U}{c_p}\right)^{-3.2} \tag{3c}$$

Note that the purely spurious correlation has a slope of −4, so that equation (2), which is spurious to some degree, has a steeper slope than (3c).

Combining (1) and (3c) and using $\Delta e/\Delta x$ as an estimate of the derivative, the fetch-dependent differential growth may be written in the form:

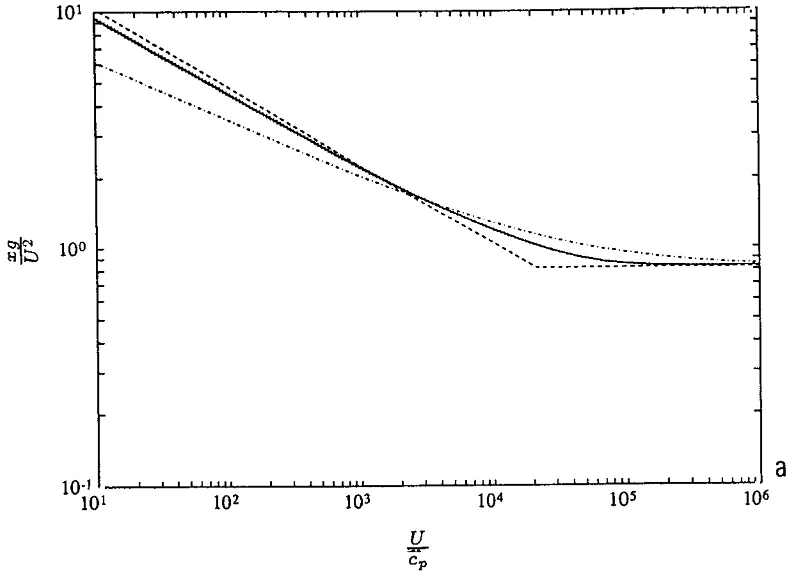
$$\frac{de}{dx} = 2\left(\frac{U}{c_p}\right)^2 \frac{U^2}{g} F_1 F_2 \tag{4a}$$

or in non-dimensional form:

$$\frac{d\tilde{e}}{d\tilde{x}} = 2\left(\frac{U}{c_p}\right)^2 F_1 F_2 \tag{4b}$$

where F_1 and F_2 are functions of U/c_p defined by the right-hand sides of (1) and (3c) and $\tilde{x}$ is the non-dimensional fetch (xg/U^2). For a steady and homogeneous wind, this is integrable in closed form:

$$\tilde{x} = 4.09 \times 10^4 \ln\left(\frac{1}{1 - 5.54\tilde{e}^{1/3.2}}\right) - 2.27 \times 10^5 (1 + 2.77\tilde{e}^{1/3.2})\tilde{e}^{1/3.2} \tag{5a}$$



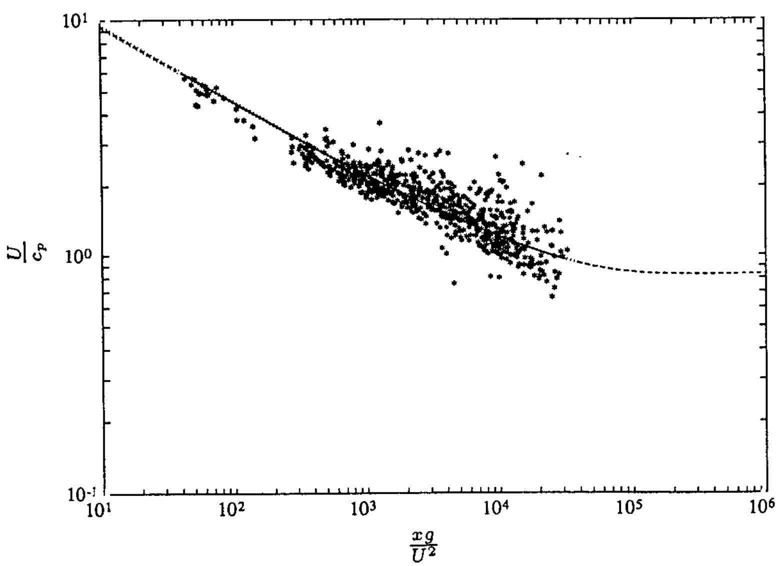


Fig. 8 Inverse wave age (U/c_p) vs non-dimensional fetch: Eq. (5b) (—) with 95% confidence limits (· · ·); JONSWAP with a Pierson-Moskowitz full development limit (– –); SMB (– · –). (b) Calculated inverse wave age (—; extrapolated: – –) and the 95% confidence limits (· · ·) showing the relationship to these data (*). (c) Calculated inverse wave age (—) showing the relationship to the JONSWAP regression line (– –) and the JONSWAP data (*).

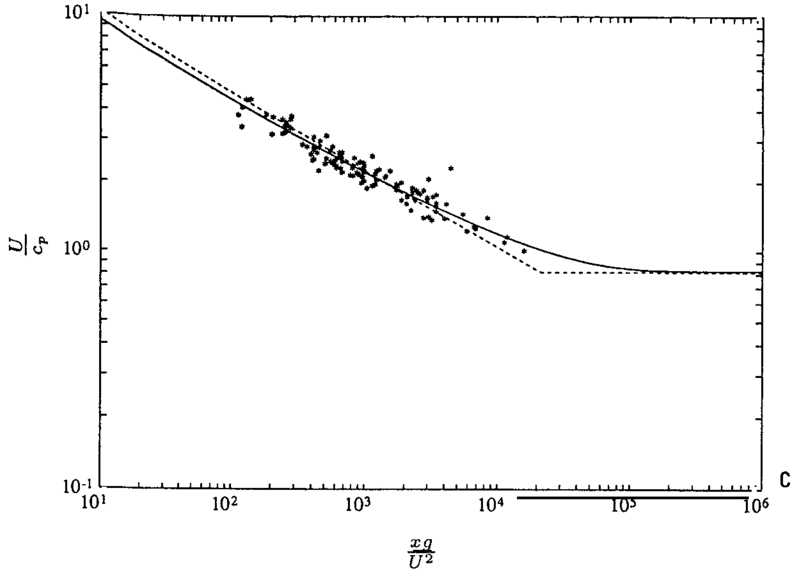


Fig. 8 (*Concluded*).

This fetch-dependence is shown on Fig. 7a, along with two well known models. The current model falls below the SMB model but is in good agreement with, although slightly above, JONSWAP at both short-fetch and full development, where they both depend on Pierson and Moskowitz (1964). In Figs 7b and c, the model is compared with the current dataset and with the JONSWAP set, respectively, and is consistent with both sets.

The non-dimensional fetch versus inverse wave age for constant wind is given by

$$\tilde{x} = 4.09 \times 10^4 \ln\left(\frac{\dfrac{U}{c_p}}{\dfrac{U}{c_p} - 0.830}\right) - 3.40 \times 10^4 \left(\frac{U}{c_p} + 0.415\right)\left(\frac{c_p}{U}\right)^2 \tag{5b}$$

The modelled inverse wave age versus fetch is shown in Fig. 8a, along with the models corresponding to Fig. 7a. Again, agreement with JONSWAP is good except in the transition region between short fetch and full development. The model is compared with the current dataset in Fig. 8b and with the JONSWAP set in Fig. 8c, and is consistent with both sets.

For unsteady or inhomogeneous winds, the fetch dependence can be derived by numerical integration of (4). The numerical procedure for integrating (4) to derive the corresponding fetch law, $e(x)$ or $\tilde{e}(\tilde{x})$ advances from the shore downwind with (3c) applied at each space step to determine the new wave age. There is some difficulty in starting the process since at the shoreline $e(0) = 0$ and $U/c_p = \infty$.

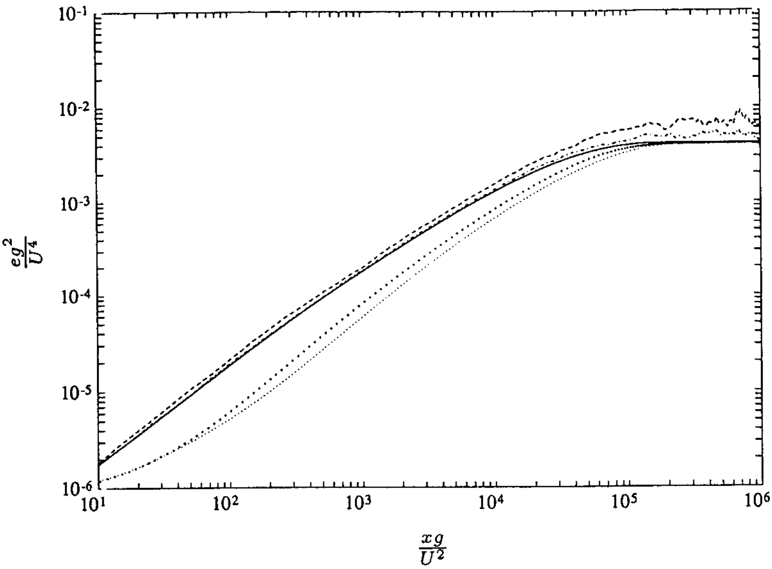


Fig. 9 Non-dimensional fetch dependence from numerical integration of Eq. (4), for constant U with fetch and three values of spatial gustiness γ: 0.0 (—); 0.2 (– · –); 0.4 (– –), and for a mean U varying with fetch according to Taylor and Lee (1984) and $\gamma = 0$: $U = 5 \text{ m s}^{-1}$ (· · · · · ·); $U = 10 \text{ m s}^{-1}$ (· · ·).

The complicated process of initiating waves at the zero fetch point is not represented by the gravity wave growth rate (1) measured here. The problem is easily avoided by using (5a) to obtain the starting energy $e(x_1)$ where x_1 is the first offshore point.

In order to simulate realistic fetch-dependent growth the wind is not held constant, but instead at each space step values are chosen at random from a Gaussian distribution of mean U and standard deviation γU. Typical values of the ratio of standard deviation to mean for the microscale are in the neighborhood of 0.1. The effect of different levels of "spatial gustiness" is explored in Fig. 9, in which fetch growth has been calculated using $\gamma = 0$, 0.2 and 0.4. These curves are obtained by averaging 50 "Monte Carlo" repetitions in each case. The principal difference in the growth curves occurs near full development and it is seen that spatial variability increases the asymptotic energy values.

If we allow the mean U to vary with fetch, according to the Taylor and Lee guidelines, we find (Fig. 9) that the growth rates are initially lower than those for constant wind but surpass those at mid-fetch and eventually reach the same full development asymptote. In each case the axes are normalized by the mean wind at full development. Furthermore, the non-dimensionalized growth rates are now wind speed dependent because the development of the internal boundary-layer over water depends on wind speed and on surface roughness, which itself is dependent on wind speed. The curves of Fig. 9 are not well represented by simple power laws and the introduction of realistic boundary-layer development at short fetch reveals a clear wind speed dependence of the non-dimensional growth rates.

4 Discussion and conclusions

The downwind development of waves from a windward shore in steady conditions has traditionally provided the most accessible data on the natural evolution of wind waves. Simple geometry and long time histories from fixed points simplify the analysis and lead, in principle, to a reproducible fetch-limited growth "law". In spite of this, various fetch-limited experiments have yielded quite different conclusions regarding the rate of development downfetch. We have argued that, among the several reasons for these differences, wind variability along fetch, boundary-layer stability and upwind shoreline geometry are prominent. Perhaps it is time to abandon the idea that a universal power law for non-dimensional fetch-limited growth rate is anything more than an idealization.

Further, it is generally accepted that at sufficiently long fetch, the wave growth rate becomes vanishingly small and a state of "full development" is asymptotically approached. If the transition from fetch-limited to full development occurs smoothly, then the slope of the fetch growth "law" takes on steadily decreasing values downfetch. This, then, is another possible cause of the differences in observed fetch growth rates, the very short fetch experiments tending to find higher growth rates than the long-fetch observations.

In this paper we have attempted to avoid the difficulties associated with fetch-limited studies by examining local differences rather than the overall fetch behaviour. In such an approach, the differences between stations are representative of the incremental growth between stations, and with increasing fetch the decrease in growth rate is readily observed. On the other hand, the statistical variability inherent in the estimation of wave properties leads to substantial scatter in the energy differences taken over relatively small fetch differences.

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References

BISHOP, C.T. and M.A. DONELAN. 1988. Waves and wave forecasting. In: *Civil Engineering Practice 3* – Geotechnical/Ocean Engineering, Technomic Publ. Co. Inc., Lancaster, Pa., pp. 653–695.

BRETSCHNEIDER, C.L. 1973. Prediction of waves and currents. Look Laboratory, Hawaii, Vol. III (1), pp. 1–17.

BURLING, R.W. 1959. The spectrum of waves at short fetches. *Dtsch Hydrogr. Z.* **12**: 45–117.

DOBSON, F.; W. PERRIE. and B. TOULANY. 1989. On the deep-water fetch laws for wind-generated surface gravity waves. ATMOSPHERE-OCEAN, **27**: 210–236.

DONELAN, M.A. 1987. The effect of swell on the growth of wind waves. *Johns Hopkins APL Tech. Digest*, **8**: 18–23.

———. 1990. Air-sea interaction. In: *The Sea: Ocean Engineering Science*, Vol. 9, John Wiley and Sons, Inc., New York, pp. 239–292.

——— and W.H. HUI. 1990. Mechanics of ocean surface waves. In: *Surface Waves and Fluxes*. Vol. 1. G.L. Geernaert and W.J. Plant (Eds), Kluwer Academic Publ., pp. 209–246.

———; J. HAMILTON, and W.H. HUI. 1985. Directional spectra of wind generated waves. *Philos. Trans. R. Soc. London*. **A315**: 509–562.

EWING, J.A. and A.K. LAING. 1987. Directional spectra of seas near full development. *J. Phys. Oceanogr*. **17**: 1696–1706.

HASSELMANN, K.; T.P. BARNETT, E. BOUWS, H. CARLSON, D.E. CARTWRIGHT, K. ENKE, J.A. EWING, H. GIENAPP, D.E. HASSELMANN, P. KRUSEMAN, A. MEERBURG, P. MÜLLER, D.J. OLBERS, K. RICHTER, W. SELL and H. WALDEN. 1973. Measurements of wind-wave growth and swell decay during during the Joint North Sea Wave Project (JONSWAP). *Dtsch Hydrogr. Z.* **A8** (Suppl.), No. 12, 95 pp.

HIDY, G.M. and E.I. PLATE. 1966. Wind action on water standing in a laboratory channel. *J. Fluid. Mech*. **26**: 651–687.

HUANG, N.E.; S.R. LONG and L.F. BLIVEN. 1981. On the importance of significant slope in empirical wind-wave studies. *J. Phys. Oceanogr*. **10**: 569–573.

JANSSEN, P.A.E.M.; G.J. KOMEN and W.J.P. DEVOOGT. 1987. Friction velocity scaling in wind wave generation. *Boundary-Layer Meteorol*. **38**: 29–35.

KAHMA, K.K. 1981. A study of the growth of the wave spectrum with fetch. *J. Phys. Oceanogr*. **11**: 1502–1515.

——— and M.A. DONELAN. 1988. A laboratory study of the minimum wind speed for wind wave generation. *J. Fluid. Mech*. **192**: 339–364.

KENNEY, B.C. 1982. Beware of spurious self-correlations! *Water Resour. Res*. **18**: 1041–1048.

KITAIGORODSKII, S.A. 1962. Applications of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process. *Bull. Acad. Sci. USSR, Geophys. Ser*. **1**: 105–117.

LIU, P.C. and D.J. SCHWAB. 1987. A comparison of methods for estimating u_* from given u_z and air-sea temperature differences. *J. Geophys. Res*. **92**: 6488–6494.

LONG, P.E., JR. and W.A. SHAFFER. 1975. Some physical and numerical aspects of boundary layer modeling. NOAA Tech. Memo. NWS TDL-56 (COM75-10980), Natl Weather Serv., Silver Springs, Md, 37 pp.

MAAT, N.; C. KRAAN and W.A. OOST. 1991. The roughness of wind waves. *Boundary-Layer Meteorol*. **54**: 89–103.

MITSUYASU, H. 1968. On the growth of the spectrum of wind generated waves. I. Rep. Res. Inst. Appl. Mech., Kyushu Univ., Vol. 16, pp. 459–482.

———. 1969, Ibid. II. Vol. 17, pp. 235–248.

MÜLLER, P. 1976. Parameterization of one-dimensional wind wave spectra and their dependence on the state of development. Hamburger Geophysikalische Einzelschriften, Heft 31, Wittenborn Söhne, Hamburg, Germany.

OKE, T. 1978. *Boundary-Layer Climates*. Methuen, London, 372 pp.

PIERSON, W.J. 1983. The measurement of the synoptic scale wind over the ocean. *J. Geophys. Res*. **88**: 1683–1708.

——— and L. MOSKOWITZ. 1964. A proposed spectral form for fully developed wind seas based on the similarity theory of S.A. Kitaigorodskii. *J. Geophys. Res*. **69**: 5181–5190.

SUTHERLAND, A.J. 1968. Growth of spectral components in a wind-generated wave train. *J. Fluid Mech*. **33**: 545–560.

SVERDRUP, H.U. and W.H. MUNK. 1947. Wind, sea and swell: Theory of relations for forecasting. Publ. No. 601, U.S. Navy Hydrogr. Off., Washington, D.C.

SWAMP (Sea Wave Modelling Project) GROUP. 1986. An intercomparison study of wind-wave prediction models. Proc. Symp. Wave Dynamics and Radio Probing of the Ocean Surface, 13–20 May 1981, Miami, Fla., Plenum Press, New York, 256 pp.

TAYLOR, P.A. and R.J. LEE. 1984. Simple guidelines for estimating wind speed variations due to small-scale topographic features. *Climatol. Bull*. **18**: 3–32.

ZWARTS, C.M.G. 1974. Transmission line wave height transducer. Proc. Int. Symp. on Ocean Wave Measurement and Analysis, 9–11 September 1974, New Orleans, La., No. 1, pp. 605–620.